TIME DOMAIN ANALYSIS OF LOSSY MULTI-CONDUCTOR TRANSMISSION LINES USING THE HILBERT TRANSFORM

Tawfik Rahal Arabi & Ricardo Suarez-Gartner

Intel Corporation
Architecture Development Laboratory
Interconnect Technology Development
5200 N.E. ElamYoung Parkway
Hilsboro, Oregon 97124.

Abstract

The objective of this paper is to present a novel approach of utilizing the Hilbert Transform to enforce the causality requirements on the electromagnetic fields in the analysis of high speed digital circuits. The technique is particularly useful in the absence of extremely accurate broad band data for the complex permittivity of lossy dielectric materials. Finally, in order to validate the accuracy and usefulness of this new approach, several numerical examples have been solved and compared to lossless and traditional non-causal models.

1. Introduction

The analysis of any electromagnetic fields problem involves the solution to Maxwell's equations in one form or another. In addition to Maxwell's equations, the solution must also satisfy the constitutive relations which describe the interaction of the EM fields with matter. The most general way to describe the constitutive relations in a linear dielectric medium is given by:

$$\vec{D} = \varepsilon \vec{E} + \varepsilon_1 \frac{\partial}{\partial t}\vec{E} + \varepsilon_2 \frac{\partial^2}{\partial t^2}\vec{E} + \ldots + \varepsilon_n \frac{\partial^n}{\partial t^n}\vec{E} \quad (1)$$

Where $\vec{D}$ is the electric displacement, $\vec{E}$ is the electric field, and ε_i is a complex constant. Assuming time harmonic EM fields, the above equation can be conveniently written in the frequency domain as:

$$\vec{D} = \varepsilon \vec{E} + \varepsilon_1 (j\omega)\vec{E} + \varepsilon_2 (j\omega)^2 \vec{E} + \ldots + \varepsilon_n (j\omega)^n \vec{E} = \varepsilon(\omega)\vec{E} \quad (2)$$

In most practical applications, equation (2) can be approximated by retaining the first term only. In this case, equation (2) reduces to:

$$\vec{D} = \varepsilon \vec{E} \quad (3)$$

It has been shown [1] that this assumption however, leads to non-causal TEM solutions to Maxwell's equations. Specifically, in a lossy homogeneous dielectric medium, equation (3) leads to non-causal TEM propagation modes. Consequently, as was shown in [1,2] an extremely accurate, broad band frequency domain characterization of equation (2) is required in order to obtain a causal model for the interaction between the electromagnetic fields and the lossy media in which they exists. Unfortunately, the required accuracy is generally not attainable by most measurement techniques and systems that appeared in the literature. In this paper, we have used the Hilbert Transform to enforce the causality requirements on the time domain electromagnetic fields in lieu of such measurement characterization.

2. The TEM Solution.

In order to illustrate the problem, without loss of generality, we consider the TEM propagation on a single lossless conductor line of infinite length and immersed in a lossy, perfectly homogeneous dielectric medium. The dielectric is characterized by a complex constant ($\varepsilon(\omega) = \varepsilon' + \varepsilon''$). It is well known that such a structure supports a TEM propagation. The transfer function for the line is given by:

OF2

CH3277-1/93/0000-0987$01.00 © 1993 IEEE

$$H(\omega) = e^{-j\omega\sqrt{\mu_0 \varepsilon(\omega)}\, l} \quad (4)$$

or, equivalently, in terms of the transmission line parameters, the above equation may be written as:

$$H(\omega) = e^{-\sqrt{[R(\omega)+j\omega L(\omega)][G(\omega)+j\omega C(\omega)]}\, l} \quad (5)$$

$$= e^{-\alpha(\omega) l} e^{-j\beta(\omega) l}$$

where l is the transmission line length, $L(\omega), C(\omega), R(\omega)$, and $G(\omega)$ are the usual per unit length frequency dependent circuit parameters of the transmission line. $\alpha(\omega)$ and $\beta(\omega)$ are therefore real functions that depend on the frequency and the line electrical and geometrical data. When the dielectric medium is described by a dielectric constant ($\varepsilon = \varepsilon' + \varepsilon''$), both $\alpha(\omega)$ and $\beta(\omega)$ reduce to linear functions of frequency [1] leading to non-causal propagation modes. The error in the time domain pulse response for this non causal model depends on the pulse rise time and the slope of $\alpha(\omega)$. In order to illustrate, we consider the example of the quasi-TEM mode of a single transmission line immersed in a perfectly homogeneous dielectric medium. For this line, the line length is 0.5 meters, the dielectric substrate height H is 5 mils, the line width W is 8.1 mils, and the dielectric constant ε is 4.0 -jε". Fig. 1 shows the pulse response of the line for different values of the dielectric dissipation factor ε". The pulse response shows a voltage at times ($t \le T_0$) where T_0 (3.28 nano seconds) is the delay of the corresponding lossless line and therefore is non-causal. It has been also shown in [2] that in order to preserve causality, the phase function in (5) must be nonlinear and very accurately known. Due to the multi-valuedness of the phase however, this implies much higher accuracy on the frequency dependence of the dielectric constant, particularly at high frequencies. Therefore, in this paper, we use the magnitude information to fully characterize the time domain response of the transmission line. The phase information is obtained directly from the magnitude information via the Hilbert Transform. Specifically, it is well known from the theory of signal processing that any causal system can be expressed as the cascade of an all pass system and a minimum phase system. For the single lossy line, the all pass system represents the lossless model of the line, while the minimum phase represents the attenuation function and thus depends on the line losses. It is also well known from the theory of signal processing that a minimum phase system exhibit the following property:

$$P_m[H(w)] = HT\{Log|H(w)|\} \quad (6)$$

where $P_m[x]$ denotes the minimum phase of the function x, $H(\omega)$ is the transfer function of the minimum phase system, and HT denotes the Hilbert Transform. Thus in order to compute the causal transfer function of the line, we compute the non-causal transfer function and use its magnitude to correct for the phase information. Specifically, the minimum phase associated with the magnitude function is calculated using equation (6). The total phase of the transfer function is then obtained as the sum of the linear and non-linear minimum phase functions. The new, causal transfer function can thus be written as:

$$H_c(\omega) = |H(\omega)| e^{-jP_m[H(\omega)]} \quad (7)$$

where $P_m[x]$ is as defined in (6). Once the causal transfer function has been obtained from (7), the causal impulse and pulse responses of the line can be obtained using standard FFT algorithms [3]. Indeed, Fig. 2 shows the causal pulse response of the line for different values of ε". It is clear from this figure that the error in the non causal model is very large, particularly for large ε"s.

3. Numerical Examples.

In this section, we consider the example of a two conductor microstrip transmission line and compute the response of the line using a lossless, a conventional lossy, and the Hilbert Transform lossy model. Although, strictly speaking, a microstrip structure does not support a purely TEM mode, we will apply the Hilbert Transform technique to each of the two quasi-TEM propagating modes. The electrical and geometrical data of the line are shown in Fig.3. The line is excited with a pulse of 50 picoseconds rise time and 100 mV magnitude. The pulse response of the line at the load end is shown in Figs. 4 & 5 for the lossless, lossy and Hilbert Transform lossy models. It is clear from these figures that the error in the non-causal model is particularly large at the load end of the excited line.

4. Summary and Conclusions.

A new technique of applying the Hilbert Transform to enforce the causality requirements on the EM fields in High Speed Transmission Line Networks has been investigated. This technique is particularly useful in the absence of highly accurate wide band data for the frequency dependent complex permittivity of the dielectric media. The new technique has been compared to the lossless and conventional lossy models on a two conductor microstrip transmission line and will be compared to measurements as well.

5. Acknowledgment.

The authors would like to thank Dr. Tapan K. Sarkar, Dr. Arthur T. Murphy, and Mr. Real Pomerleau for several enlightening discussions.

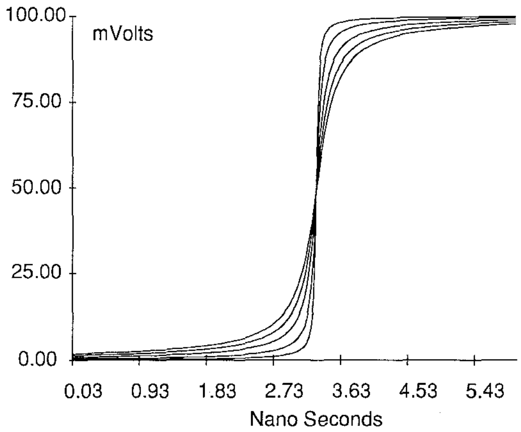
6. References.

[1] Tawfik Rahal Arabi, Arthur T. Murphy, Tapan K. Sarkar, Roger F. Harrington, and Antonije R. Djordjevic, " On the Modeling of Conductor and Substrate Loss in Multi-Conductor Transmission Lines," To be published in **IEEE Trans., Microwave Theory Tech.,** July 1991.

[2] Tawfik Rahal Arabi, Arthur T. Murphy, Tapan K. Sarkar, and Antonije R. Djordjevic, " An Efficient Time Domain Technique For The Analysis of Multi-conductor Transmission Lines Using The Hilbert Transform," **in MTT-S International Microwave Symposium.** June 1991.

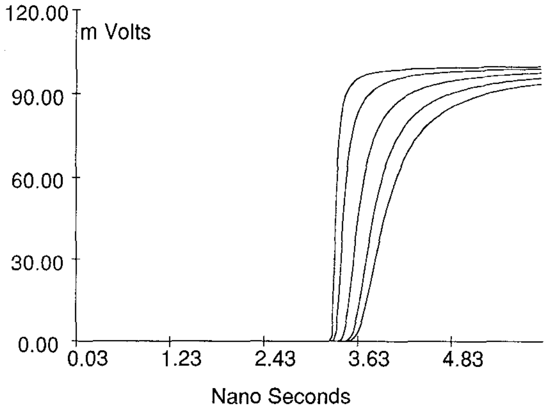
[3] Antonije R. Djordjevic, Tapan K. Sarkar and Roger F. Harrington "Analysis of Lossy Transmission Lines with Arbitrary Nonlinear Networks," **IEEE Trans. Microwave Theory Tech.,** Vol. MTT-34, pp. 660-666, June 1986.

[4] Cao Wei, Roger F. Harrington, Joseph R. Mautz and Tapan K. Sarkar "Multiconductor Transmission Lines in Multilayered Dielectric Media," **IEEE Trans. Microwave Theory Tech.,** Vol. MTT-32, pp. 439-450, April 1984.



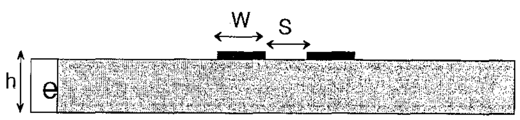
$\varepsilon_1'' = 0.048, \varepsilon_2'' = 0.12, \varepsilon_3'' = 0.24, \varepsilon_4'' = 0.36, \varepsilon_5'' = 0.48$

Fig. 1: Non-Causal Pulse Response For Different Values of ε"



$\varepsilon_1'' = 0.048, \varepsilon_2'' = 0.12, \varepsilon_3'' = 0.24, \varepsilon_4'' = 0.36, \varepsilon_5'' = 0.48$

Fig. 2: Causal Pulse Response For Different Values of ε"



h=20mils, W=20mils, S=10 mils, ε' = 4.5, ε" = 0.05.

Fig. 3: Two Conductor Microstrip Line.

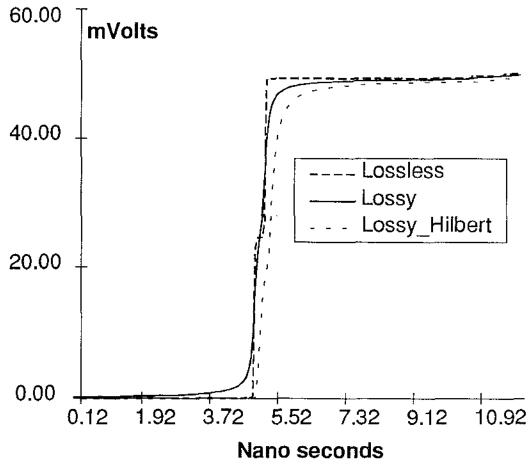


Fig. 4: Response of The Two Conductor Line of Fig. 3 at Load End. Line 1 Excited, Line 1 Observed Excitation Pulse: 50 ps Rise Time, 100 mV Magnitude.

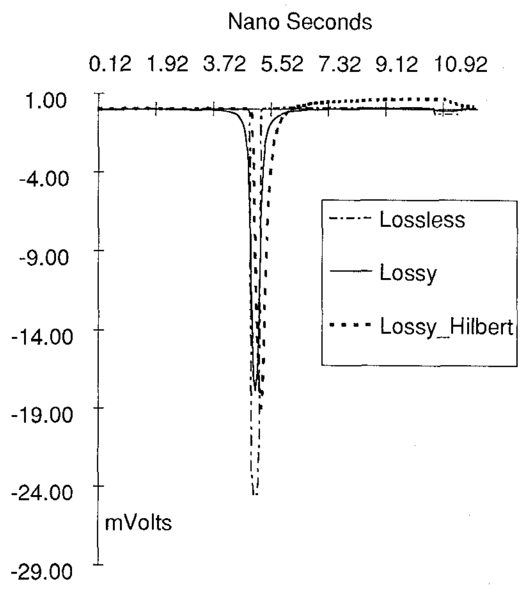


Fig. 5: Response of The Two Conductor Line of Fig. 3 at Load End.
Line 1 Excited, Line 2 Observed.
Excitation Pulse: 50 ps Rise Time, 100 mV Magnitude.